CSC 238 2000 (Day section)

St. George Campus | University of Toronto

MIDTERM EXAM

Do not open this booklet until you are told to do so.

Last Name:

First Name(s):

Student number:

Tutor's name:

There are 4 questions for a total of 50 marks. The total time is 50 minutes.
The exam is closed book, and no aids are allowed.
Answer directly on the paper in the spaces provided or on the 2 blank pages at the end.
For rough work, you may use the backs of pages. *These will not be marked.*
There should be 7 pages in this booklet, including this one and the blank ones.

Good luck!

Question 1.	/ 10
Question 2.	/ 15
Question 3.	/ 10
Question 4.	/ 15
Total	/ 50

1. (10 Marks) Prove by simple induction that for all $n \in \mathbb{N}$ such that $n \geq 2$, $5^n \geq 3n$.

2. (15 Marks) Consider a function f from $\mathbb{N}$ to $\mathbb{N}$ satisfying

$$
\begin{aligned}
&f(0) = 1,\\
&f(1) = 3,\\
&f(n) = f(n-1) + 2f(n-2), \text{ for all } n \geq 2.
\end{aligned}
$$

Prove using complete induction that for all $n \in \mathbb{N}$, $f(n) \leq 3^n$.

3. (10 marks) Prove from first principles (that is, without using any rules about bounds of sums, products, *etc.*) that $f \in \mathcal{O}(n^2)$, where $f(n) = 3n^2 - 2n + 8, \quad \text{for all } n \in \mathbb{N}$.

4. (15 Marks) Consider the following program, where A is an integer array:

$$
\begin{array}{l}
index := 0; \quad count := 0 \\
\textbf{while } (index \neq m) \textbf{ do} \\
\quad index := index + 1 \\
\quad \textbf{if } (A[index] \neq 0) \textbf{ then} \\
\quad\quad count := count + 1 \\
\quad \textbf{end if} \\
\textbf{end while}
\end{array}
$$

(a) State a loop invariant lemma for this program. You do not need to prove this lemma, but it should be strong enough to allow you to complete part (b).

(b) Prove (using the lemma from part (a)) the partial correctness theorem for this program for the following precondition / postcondition pair:

Precondition: m is a positive integer and A has elements $A[1], \ldots, A[m]$.

Postcondition: $count = |\{i : 1 \leq i \leq m \text{ and } A[i] \neq 0\}|$.
(In other words, *count* is the number of elements of A that are non-zero.)

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